MID-ALTITUDE NAVIGATION SATELLITES

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ABSTRACT - After considering both lower and higher altitudes the mid-altitude (approximately 1 earth's diameter) polar circular satellite constellation has been selected as a prime possibility for an accurate, all weather, always available, three dimension, U.S. based navigation system. The system designed is based upon using the well-known low altitude satellite techniques to extrapolate this knowledge to higher altitudes.

At the higher altitudes the drag and gravitational anomalies decrease rapidly but the knowledge of satellite position has a minimum at about one earth radius. The combination of gravitational anomalies and knowledge of satellite position shows a rather broad minimum for satellites having periods between 8 and 24 hours.

At present the principal parameter which favors a lower period is the satellite clock, the principal factor favoring higher altitudes is coverage.

The number of satellites visible to an observer at the equator for three planes of 8 satellites per plane is shown for a portion of one day and the background information found from a low altitude satellite is shown and discussed.

This paper discusses the genesis of a mid-altitude system for continuous 3 dimension navigation. This system developed after a look was taken at both lower and higher constellations. Among those studied were the 105 min, 2 hr, 4 hr, 6 hr, 8 hr, 12 hr and 24 hour circular orbits.

After these studies were made it was found that it is possible to minimize the number of satellites and the number of ground stations. The number required is not changed if all satellites are polar and with polar one can control all such satellites with a single ground station. For the best accuracy other ground stations may be needed but the minimum remains at one.

For reasonable minimum elevation angles (10°) the number of satellites required is constant as one increases satellite altitude above 6500 miles until an altitude of 14,000 miles is reached. The reason that the number remains constant is that the radius of coverage increases very slowly with altitude above this altitude. Figure 1 shows the relationship. The recommended altitude depends on a number of small effects and a few larger ones.

Before discussing them it would be well to spend a little time discussing the low and the synchronous altitude constellation.

The low altitude (600 mi.) orbit is valuable because its characteristics are well known and because this constellation has been successfully providing intermittent navigation of submarines and ships. Since the characteristics of this orbital altitude are so well known they form a useful anchor point for alternative constellations. However, the 600 mile orbit requires more than 100 satellites to have three satellites visible at all times. On the plus side only one ground station is needed for low altitude polar satellites.

The next satellite habitat looked at is the intuitively prominent equatorial (quasi stationary) satellites. The advantage of such satellites is that they are more or less fixed in space and, as such, they are analogous to fixed ground radio stations on earth. The satellite has the advantage that it has a direct line of sight to the navigator and can therefore use frequencies little affected by the propagation path.

The principal advantage of the synchronous satellite is also its principal disadvantage. Since such a satellite is quasi fixed it does not scan the ground stations and several stations must be used to find how quasi the quasi fixed objects are. Another problem is that satellites existing over unfriendly territory are always visible to the unfriendly schemes of the opposition. In addition this constellation provides poor latitude determination at the equator and no coverage at the poles.

For a single "stationary" synchronous satellite it is well to use four ground stations suitably located in northern and southern hemispheres to define satellite navigation reference position relative to earth. A line of satellites about the equator requires 2 lines of ground stations and the ground station problem soon becomes immense. United States possessions will cover nearly 180° of the earth in the northern hemisphere but nothing suitable in the southern hemisphere.

At this point we decided to take a closer look at the geometry of the situation. The first item that should be decided is the minimum elevation angle that will be used. Two principal factors are present. The tropospheric refraction varies rapidly at low elevation angles and the ionosphere refraction varies very little with angle, as shown in Fig. 2. Since the cutoff angle is somewhat arbitrary we will use the figure of 10° for the vigator and 5° for the ground station with the ssumption that the ground station complications are worth instrumenting for the additional coverage while the navigator should not be so impeded.

The layout of ground stations can be described as a static constellation while satellites are dynamic constellations. What is said about one applies somewhat to the other; one can look at both satellite and ground station constellations simultaneously.

For instance, the geometric minimum number of satellites or ground stations that can provide coverage (by one satellite or ground station) throughout the world is four. To have such coverage by 4 satellites they must be at or near the apices of a tetrahedron and for a minimum of 10° elevation angle they must be above 25,000 miles. The problem that makes a constellation unusable for satellites is that such a dynamic constellation relationship cannot be realized. The ground station complex is usable provided one can find suitable politically stable locations and the satellites are high enough to be seen for the required portion of the orbit.

The practical minimum number of satellites that will provide at least one satellite visible everywhere from a stable configuration is six, three in each of two planes. For such satellites the minimum altitude required is 6500 miles to provide a minimum elevation angle of 10°. To have three satellites visible it is sufficient to triple this number to 18. The minimum number necessary to have three visible above 10° elevation angle appears to be 12.

We can summarize this discussion by saying that the minimum number of satellites required to have 3 in view at all times is approximately 12 and the minimum number of ground stations required to have all the satellites in view is four. Since the same geometrical argument is used for both the requirement to have a minimum number of satellites and a minimum number of ground stations can be satisfied simultaneously. (Though at the lower altitudes the four ground stations will not see the satellites at all times.)

Optimum Satellite Altitude

The principal errors in the knowledge of position of the 600 mile satellites are those due to drag and to unknown gravitational anomalies. At 6500 miles the drag is reduced by a factor of more than one million from that at 600 miles. It becomes completely negligible. The gravitational anomalies are expected to decrease as shown in Fig. 3. The effect of measurement errors is least where the distance between stations is roughly equal to the height of the object being located. On the earth's surface the maximum distance between ground stations is roughly an earth radius so the results of Fig. 4 are not surprising. The sum of gravitational effects plus instrumentation errors is shown in Fig. 5. Here it is seen that a broad error minimum occurs between an eight and a twenty four hour orbit.

The cost of putting up a 24 hour satellite is little greater than for an 8 hour object, the station keeping equipment is more costly and the altitude keeping equipment is also more massive so, while these three items are relatively small they all tend to favor a lower constellation. The largest factor that favors the lower constellation is other than these - it is the stability of the satellite oscillator as referred to the ground station.

The Navigation System

Nearly all radio navigation systems are time oriented in either a spherical or hyperbolic sense. In either case the satellite must have a stable time source. The choice of a time source is limited to either a quartz oscillator or an atomic standard. It appears that a quartz oscillator may have the required stability for a 4 hour drift time but that a 6 hour drift time may make it necessary to have an atomic oscillator. The attendant weight, complication and power needed for the atomic oscillator again tends to favor a lower altitude, where the quartz oscillator has a better chance of being satisfactory and, even if not, either oscillator will incur less error.

The Constellation

Previously we have shown that the minimum number of satellites necessary to have three visible everywhere on the earth's surface is approximately 12. We can look at the constellation again with the idea that perhaps fewer satellites could be used or perhaps a few more would do the job better. We have not found that fewer satellites are satisfactory. Further, we have found that a mix of inclinations provides negligible improvement over having all polar. We do find that a few more satellites does improve the system. The principal reason that more satellites are desirable is that, while three satellites are enough for navigation, occasionally one must have a fourth in order to correct one's clock.

With polar satellites the equatorial regions are those that are covered least. Accordingly, coverage at the equator will determine the worst case coverage. Figure 6 shows the coverage for an observer at the equator as the satellites change throughout the first six hours of the day. Satellites are shown only if their elevation angles are above 10°. It is seen that a minimum of four are available over this period. Accordingly, we can assume that a three plane, 8 satellites per plane constellation is sufficient and the loss of one or two satellites in such a constellation is not catastrophic.

Only one item remains in this little study. We now have an idea of the number of satellite planes, the number of satellites per plane, the inclination of the planes and the altitude of the satellites. The one item remaining is the orbital eccentricity. One problem related to eccentricity, assuming a stable satellite clock, is that of changes in the clock frequency due to relativity. The clock frequency changes and the phase output integrates these changes due to the clock experiencing a varying gravitational field. Figure 7 shows the

the magnitude of this effect versus orbit eccentricity. It is seen that, depending on the accuracy desired, the relativity effect can be significant. We accordingly recommend a circular orbit.

Previously it has been stated that this mid-altitude proposal is based on experience obtained from satellites at lower altitudes. Such a low altitude satellite has been in orbit for two years and has shown that the concept of a time oriented satellite navigation technique is feasible.

Figure 8 shows the measured stability of the oscillator in orbit. It is seen that the oscillator drift is small - approximately 2.5 parts in 10^{12}/day but that the temperature coefficient is large - approximately 2pp10^{11}/degree C. Figure 9 shows the results obtained by using the satellite as a time transfer device between an oscillator at NRL kept reasonably close to the Naval Observatory standard and cesium beam resonators at the four stations shown.

By making the measurements when the satellite is near its closest approach to the station the main error in satellite position, the along track error, is eliminated. No place on earth is more than one hour separated from a station near the pole and the ride is ideal from a vibration standpoint. Used properly the low altitude circular satellite is a near ideal time transfer agent.

This paper can be summarized as showing that the best worldwide satellite navigation system will be obtained from the use of a constellation of circular, polar, mid-altitude satellites.

Acknowledgements

The work of many other persons is present in this report. While all cannot be mentioned I should single out Mr. Ralph Brescia as having made major contributions.

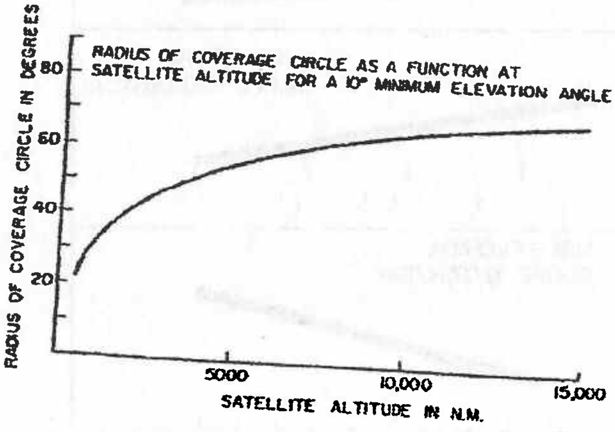

RADIUS OF COVERAGE CIRCLE AS A FUNCTION AT SATELLITE ALTITUDE FOR A 10° MINIMUM ELEVATION ANGLE

Fig. 1

PHASE DELAY IN IONOSPHERE AND TROPOSPHERE AS A FUNCTION OF ELEVATION ANGLE

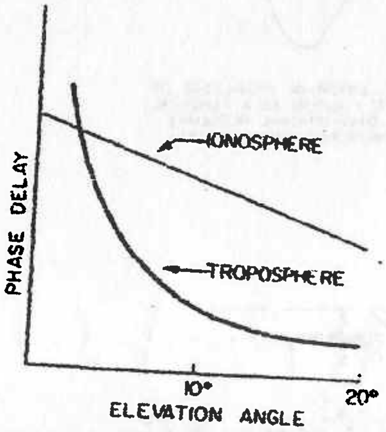


Fig. 2

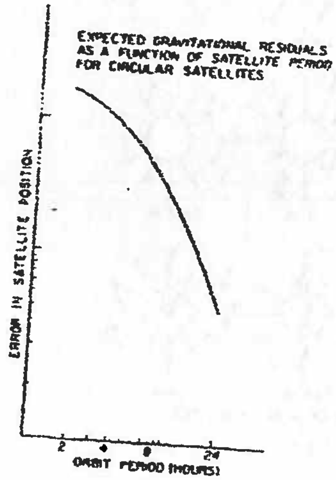

EXPECTED GRAVITATIONAL RESIDUALS AS A FUNCTION OF SATELLITE PERIOD FOR CIRCULAR SATELLITES

FIG 3

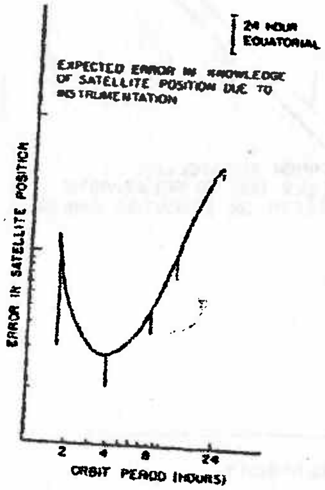

EXPECTED ERROR IN KNOWLEDGE OF SATELLITE POSITION DUE TO INSTRUMENTATION

FIG 4

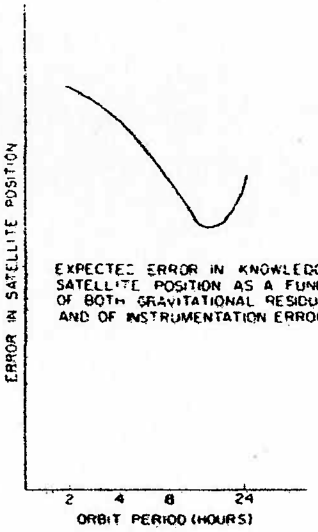
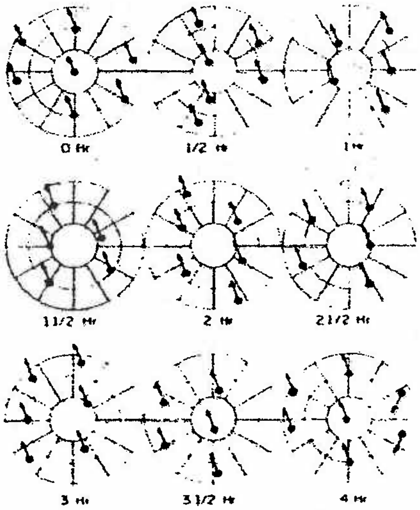


Fig. 5



3x8 CONSTELLATION OF 12 Hr SATELLITES

Fig 6

669

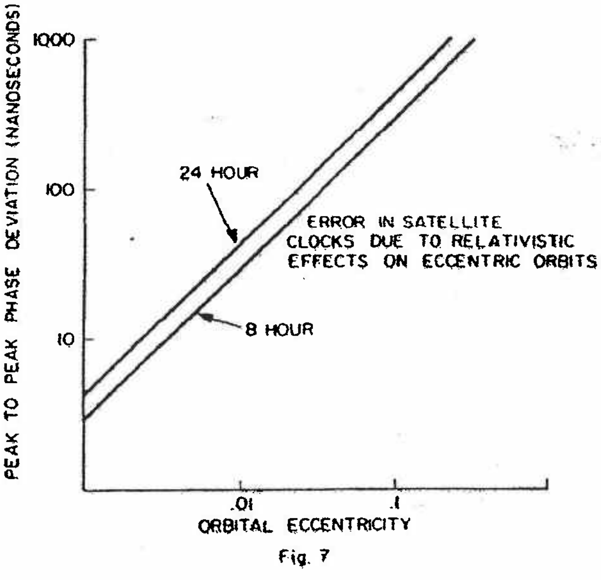


Fig. 7

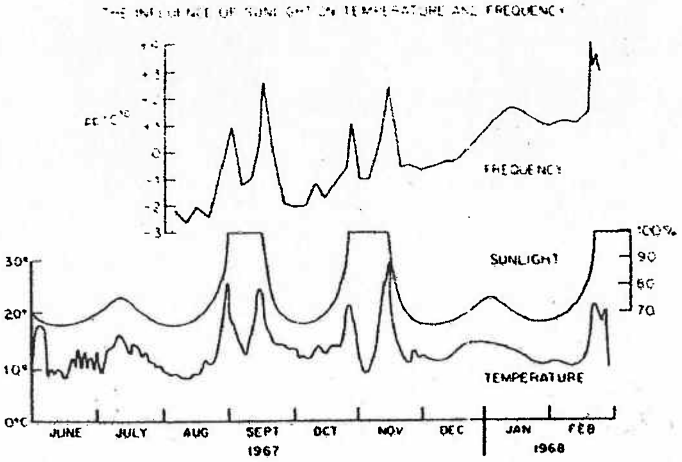


Fig. 8

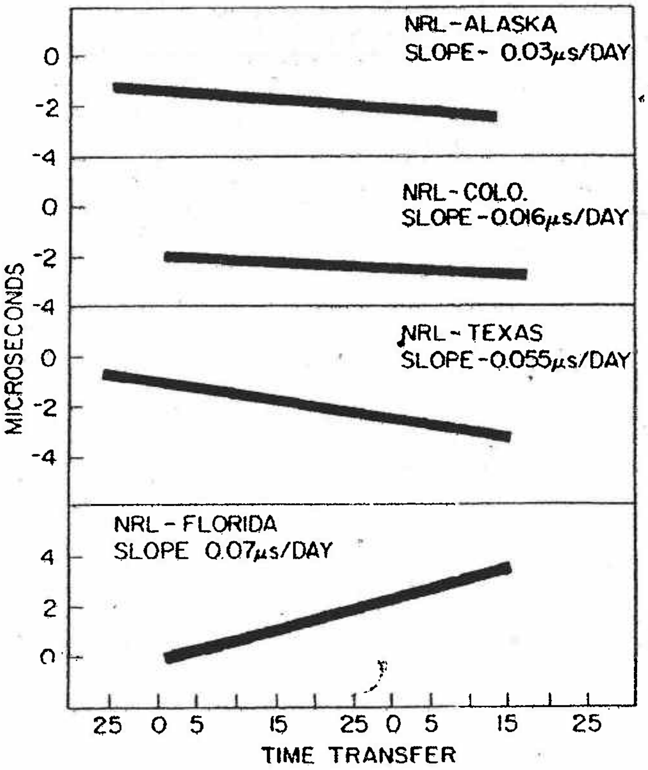


Fig. 9